# Pion polarizability from the lattice

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# Outline

- Hadrons in the magnetic field on the lattice
- Spin degrees of freedom
- Vector meson magnetic moments
- Pion polarizability
- Conclusions

# Lattice QCD in magnetic field

- Lattice – well grounded NPQCD method
- Numerous applications to hadron (spin) structure – talk of M. Deka
- Strong magnetic field – for Heavy-Ion Collisions

$$A_{\mu\, ij} \to A_{\mu\, ij} + A^B_\mu \delta_{ij},$$

$$D\psi_k = i\lambda_k \psi_k, \quad D = \gamma^\mu(\partial_\mu - iA_\mu)$$

$$A^B_\mu(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1}).$$

# Magnetic field and mesons (cf talk by M. Deka)

- Correlator $\langle \psi^\dagger(x) O_1 \psi(x) \psi^\dagger(y) O_2 \psi(y) \rangle_A,$

$$\langle \bar\psi O_1 \psi \bar\psi O_2 \psi \rangle_A = -\mathrm{Tr}\,[O_1 D^{-1}(x,y) O_2 D^{-1}(y,x)]$$
$$+\mathrm{Tr}\,[O_1 D^{-1}(x,x)]\,\mathrm{Tr}\,[O_2 D^{-1}(y,y)]$$

- Disconnected is absent for isovector currents
- Full set of hadronic states

$$\tilde C(n_t) = \langle \psi^\dagger(\mathbf{0}, n_t) O_1 \psi(\mathbf{0}, n_t) \psi^\dagger(\mathbf{0}, 0) O_2 \psi(\mathbf{0}, 0) \rangle_A = \sum_k \langle 0|O_1|k\rangle \langle k|O_2^\dagger|0\rangle e^{-n_t a E_k},$$

$$\tilde C(n_t) = A_0 e^{-n_t a E_0} + A_1 e^{-n_t a E_1} + \ldots\ ,$$

$$\tilde C_{fit}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} = 2 A_0 e^{-N_T a E_0/2} \cosh\left(\left(\frac{N_T}{2} - n_t\right) a E_0\right).$$

# Spin in magnetic field

- Components of correlators

$$C^{VV}_{xx} = \langle \bar{\psi}(\mathbf{0}, n_t)\gamma_1\psi(\mathbf{0}, n_t)\bar{\psi}(\mathbf{0}, 0)\gamma_1\psi(\mathbf{0}, 0)\rangle$$

$$C^{VV}_{yy} = \langle \bar{\psi}(\mathbf{0}, n_t)\gamma_2\psi(\mathbf{0}, n_t)\bar{\psi}(\mathbf{0}, 0)\gamma_2\psi(\mathbf{0}, 0)\rangle$$

$$C^{VV}_{zz} = \langle \bar{\psi}(\mathbf{0}, n_t)\gamma_3\psi(\mathbf{0}, n_t)\bar{\psi}(\mathbf{0}, 0)\gamma_3\psi(\mathbf{0}, 0)\rangle$$

- Rho density matrix

$$C^{VV}(s_z = \pm 1) = C^{VV}_{xx} + C^{VV}_{yy} \pm i(C^{VV}_{xy} - C^{VV}_{yx})$$

# Magnetic moments and polarizabilities

- Shifted Landau level

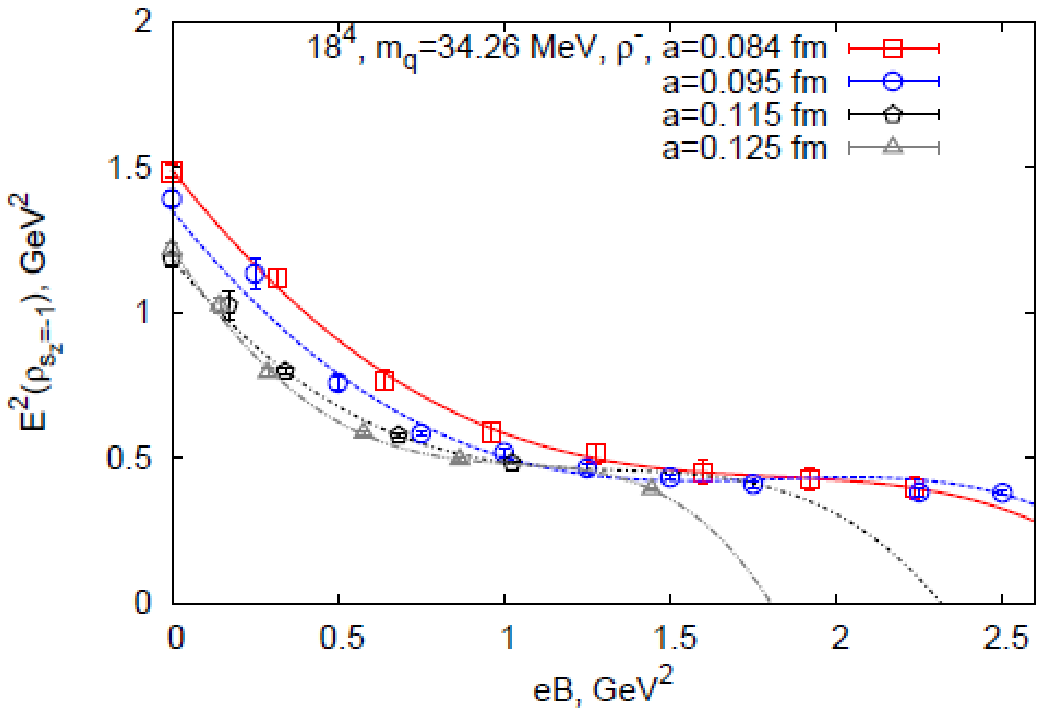
$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta (qB)^2 + k(qB)^4$$

- Small fields – g-factor: 2.4(0.2)

$$g = \frac{E^2(s=+1) - E^2(s=-1)}{2(eB)}.$$

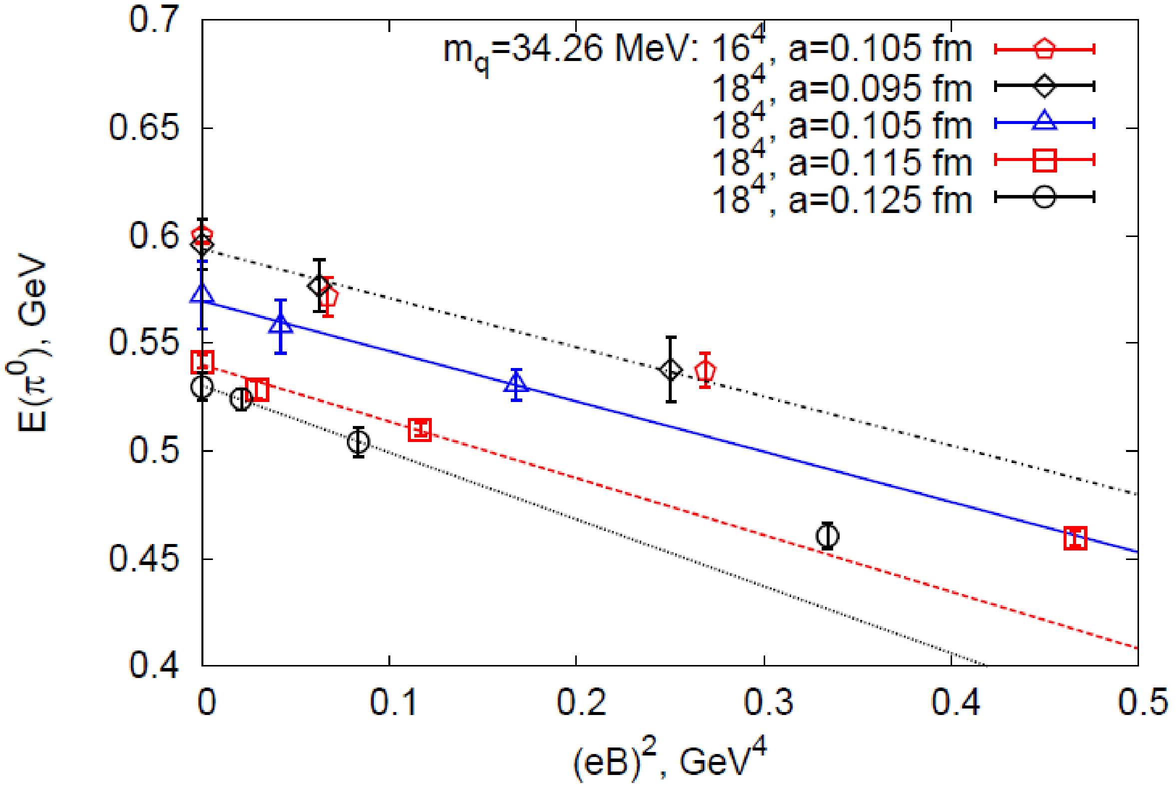
# Small and large fields

- No tachyonic mode! NPQCD don't like vacuum supercondactivity



# Magnetic polarizability for pion: neutral – simpler (no Landau level shift term)

- Small fields $E = E(B=0) - 2\pi\beta_m(eB)^2$



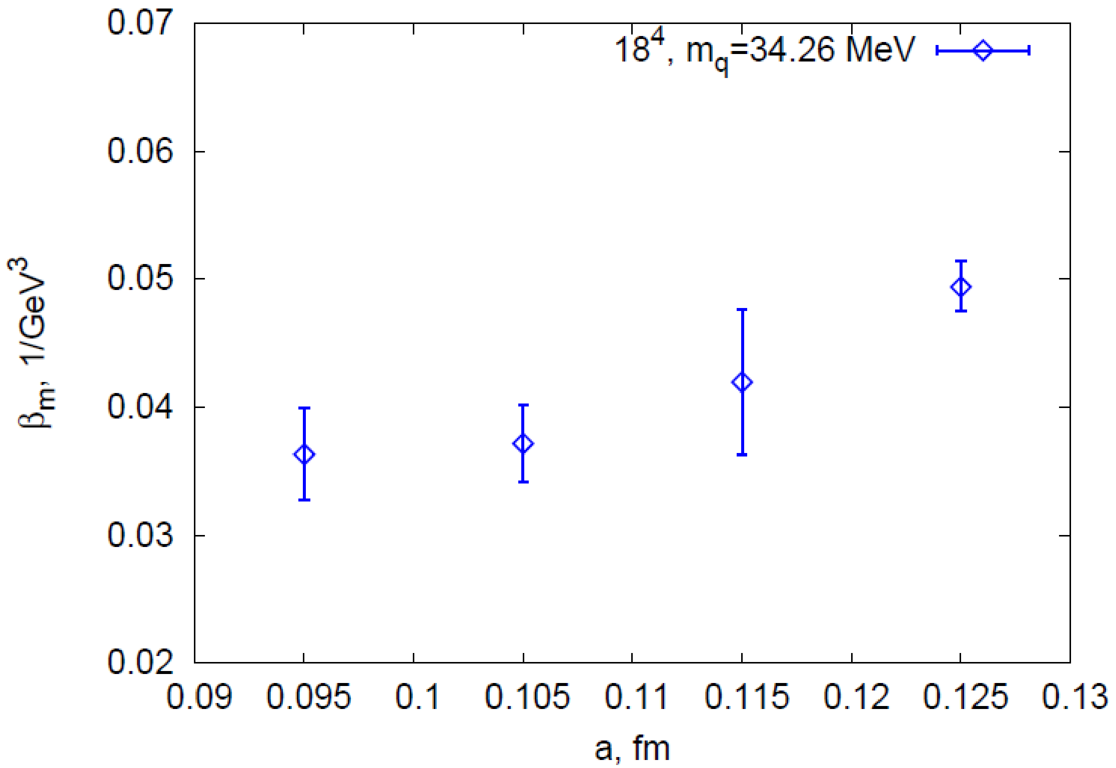
# Polarizability

- Dependence on lattice parameters

| $V_{latt}$ | $a\ (fm)$ | $\beta_m^{m_q=34\ MeV}$ (GeV$^{-3}$) | Error (GeV$^{-3}$) | $\chi^2$/d.o.f. |
|---|---|---|---|---|
| $18^4$ | 0.095 | 0.036 | 0.004 | 0.0915051 |
| $18^4$ | 0.105 | 0.037 | 0.003 | 0.0501177 |
| $18^4$ | 0.115 | 0.042 | 0.006 | 1.03419 |
| $18^4$ | 0.125 | 0.049 | 0.002 | 0.0130633 |

- Inverse GeV -> fm – divide by ~$5^3$=125

# Lattice spacing dependence



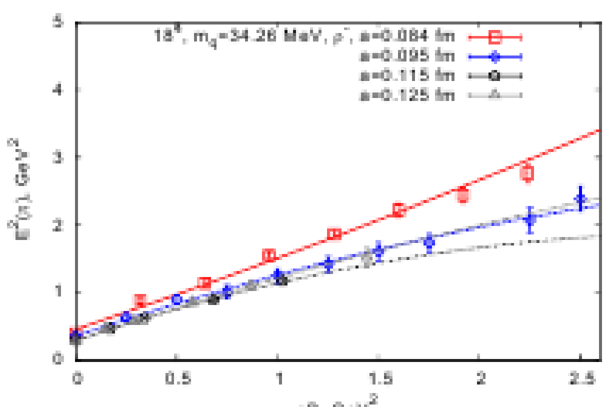
# Comparison with ChPT (M. Ivanov)

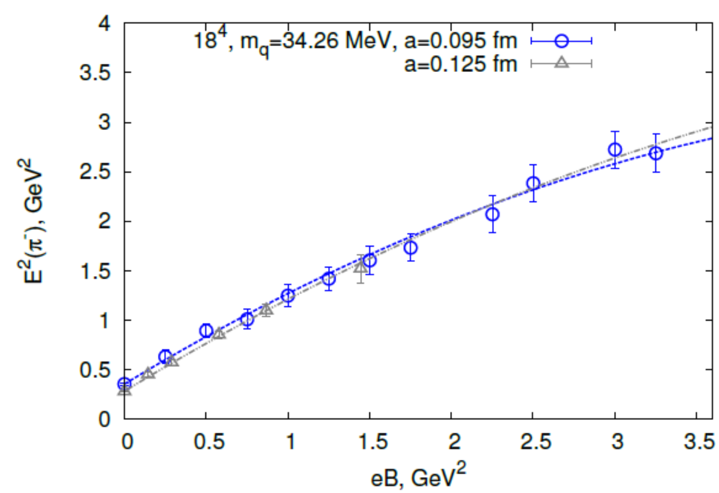
- More close to lattice at 2 loops:
- 0.5 (1 Loop) -> 1.5 (2 Loops) -> 2.5(Lattice)

| | ChPT to one loop | ChPT to two-loops |
|---|---|---|
| $(\alpha - \beta)_{\pi^0}$ | $-1.0$ | $-1.9 \pm 0.2$ |
| $(\alpha + \beta)_{\pi^0}$ | $0$ | $1.1 \pm 0.3$ |

# Charged pions: measured by COMPASS (talk of J. Friedrich)

- More difficult because of Landau level shift
- Strong dependence on lattice spacing
- First preliminary high statistics data obtained



# Charged pion magnetic polarizability

- Scale at 1 fm level
- Possible (likely)? role of higher-nonlinearities
- More statistics required
- Coming (hopefully) soon….

# Conclusions/Outlook

- Lattice QCD is capable to calculate (nonlinear) polarizabilities from low to large fields (searching for vacuum superconductivity)
- Reasonable compatibility with ChPT
- Kaons in progress