Exclusive pion-nucleon Drell-Yan processes

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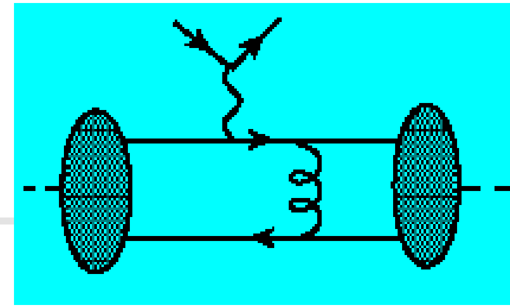
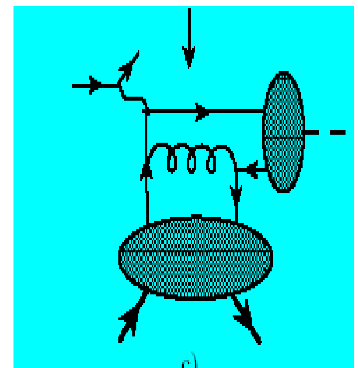
Outline

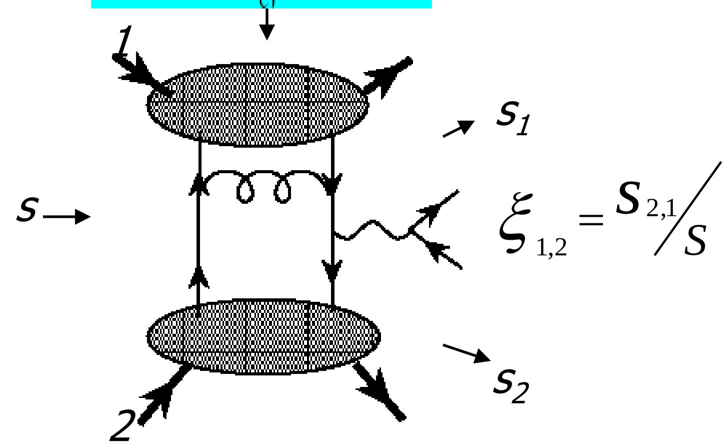
- DY&GPD: exclusive DY, analyticity and factorization
- Interference between exclusive lepton pair production mechanisms
- Transverse SSA in DY: contour gauge and factor 2
- DYW/BG-type duality in DY: SSA, Sivers function and time-like formfactors

Ways to ,
to exclusive DY

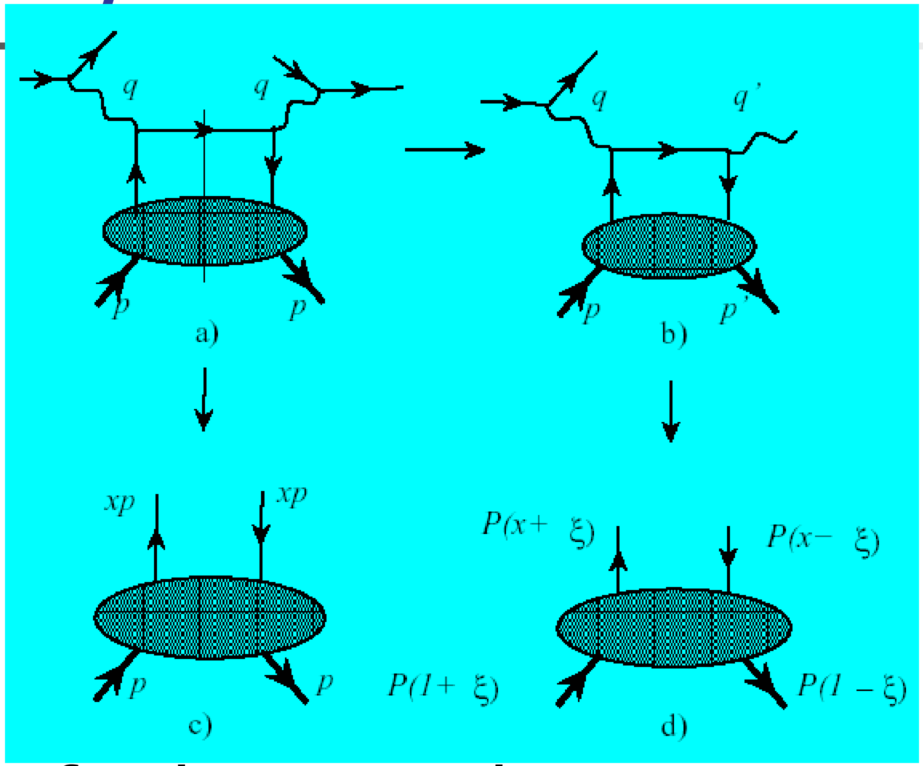




- Simplest case - pion FF(ERBL)
- Change DA to GPD (talks of K. Kumericki, S.Goloskokov, O. Kouznetsov) - exclusive electroproduction
- $M_{DY} \sim M_{DVCS} F_{pi\ g\ g^*}$
- Time from right to left- exclusive DY (DAxGPD)- Berger,Diehl,Pire
- Phase sign change: c.f. Sivers for SIDIS/DY
- Analytic properties ~ factorization (not completely lear yet)
- Second DA->GPD-another mechanism- OT'05 (problems with factorization -analytic continuation to be performed)



QCD Factorization for DIS and DVCS/DVMP



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – if $1 \leq |x_B|$ polynomial in $1/x_B$

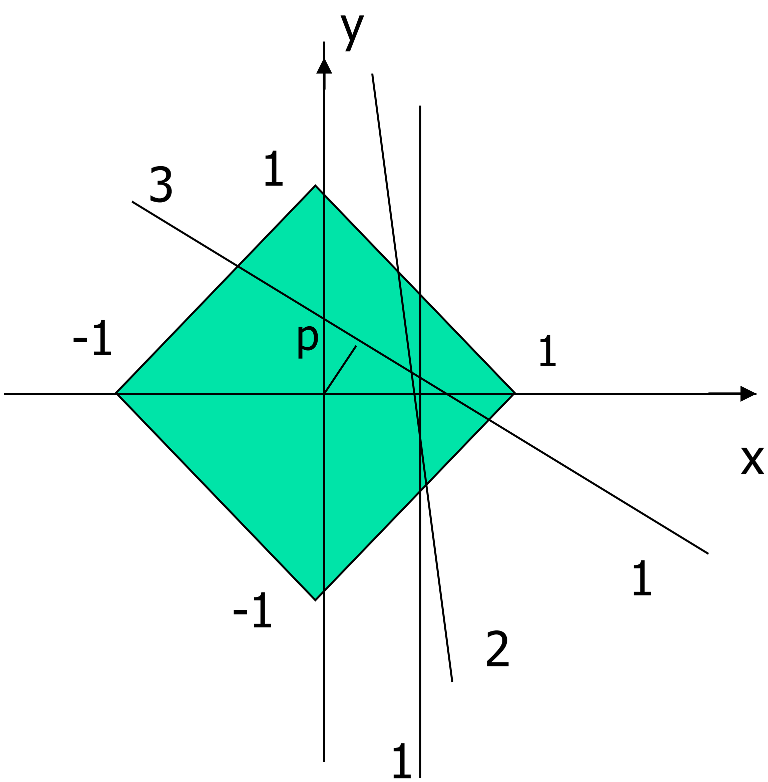
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$ ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$ - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

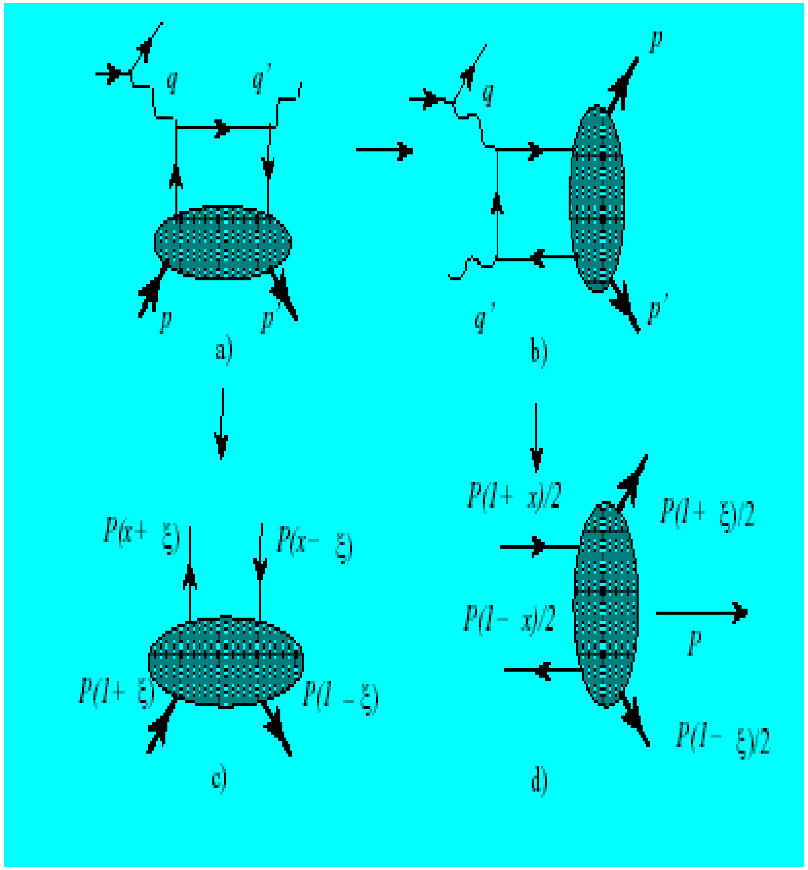


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi |cos\phi| (H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in *x* weighted with x^n - are polynomials in $1/\xi$ of power *n+1*
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity ("dynamical") -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

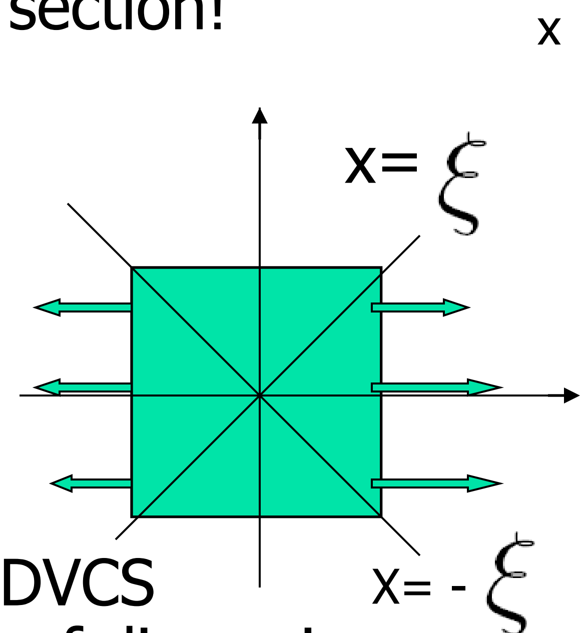
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x-\xi+i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- ERBL ⟶ "GDA" region
- Strategy (now adopted) of GPD's

studies: start at diagonals

(through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

DY?!

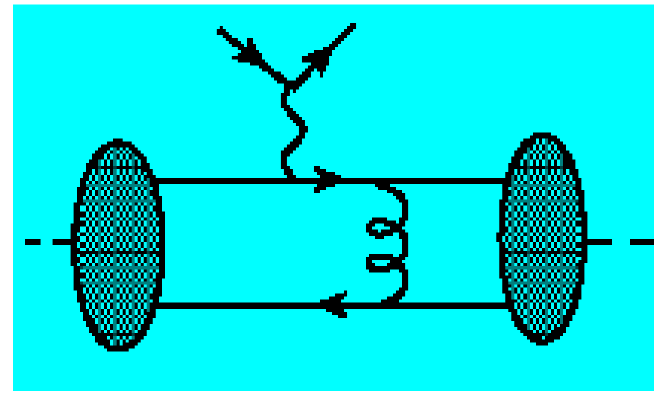
Time-like amplitudes and analyticity

- Extra cuts in Q^2 appear
- Scaling: $F(Q^2/s) = F(-Q^2/-s)$: cuts cancellation (cf inclusive electron-positron annihilation)
- $F(Q^2/u) = F(-Q^2/-u)$ – cuts in Q^2 has a form of cuts in u (opposite pole prescription) – diagonals on holographic plot interchanged
- Factorization proof: start in unphysical region in skewness where factorization holds (cuts cancelled but what about NLO?! – nonfactorizable corrections!?) and continue to physical region
- Problems with space-time picture (J.W.Qiu)

“Dispersive” factorizaton proof

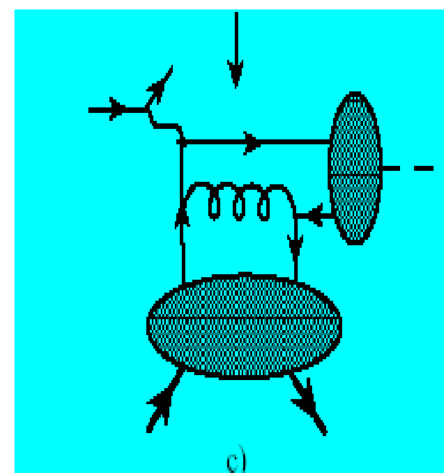
- Starting from (Pion) form factor- 2 DA’s –no cuts

$$F \sim \left(\int dx \frac{\phi(x)}{1-x}\right)^2$$



- 1 DA -> GPD :Exclusive mesons production: Factorization = DR + D-subtraction
- (DVMP/DY) - +/-

$$M \sim \int dx \frac{\phi(x)}{1-x} \int dx \frac{H(x,\xi)}{x-\xi+i\varepsilon}$$



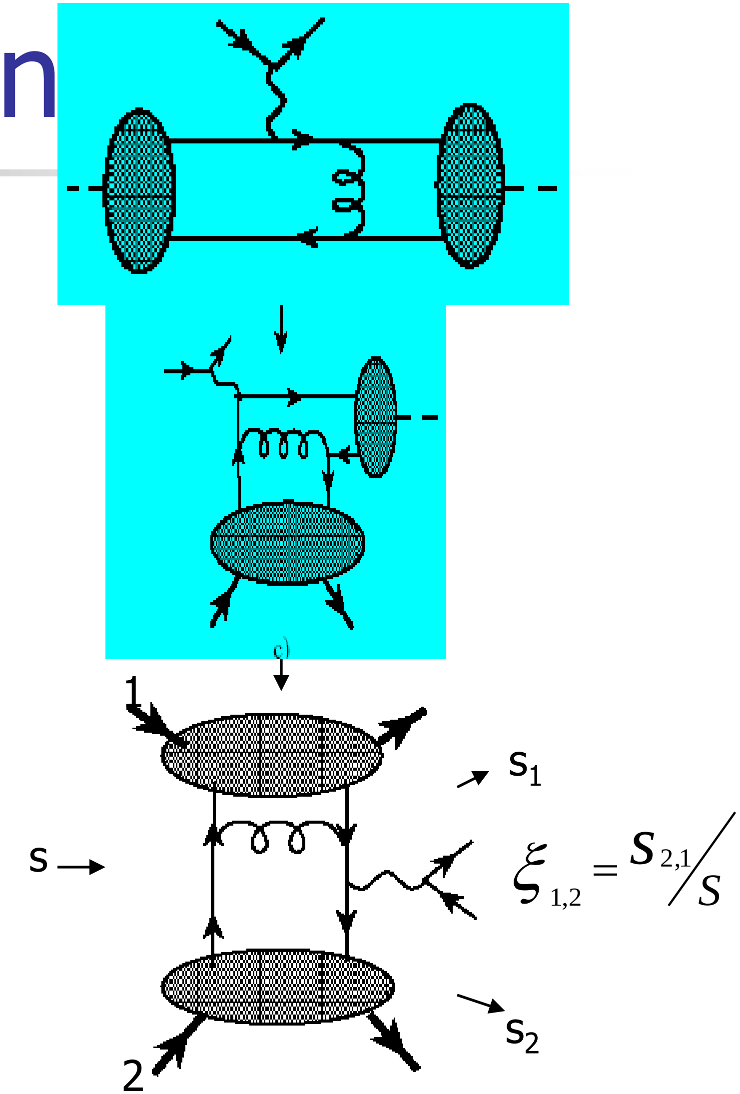
Next step: 2 DA's -> 2 GPD's- Double Diffraction

- Exclusive double diffractive DY process
- Analytic continuation:

$$M \sim \int dx \frac{H(x,\xi_1)}{x-\xi_1 \pm i\varepsilon} \int dy \frac{H(y,\xi_2)}{y-\xi_2 \mp i\varepsilon}$$

- DIFFERS from direct calculation – NO factorization in physical region

$$M \sim \int\int dxdy \frac{H(x,\xi_1)H(y,\xi_2)}{(x-\xi_1)(y-\xi_2)+i\varepsilon}$$



Double Diffraction: properties and problems

- Holographic equation: DR contains double and single (linear in D-term) dispersion integrals as well as subtraction (quadratic in D-term)
- $F(Q^2/s) \to F(s_i/s), F(s_i/s_j)$
- Analytic continuation for

various cuts is still unclear. Possible cancellation of cuts – real amplitude?

Analitycity for HT(and k_T)

Kinematical regions

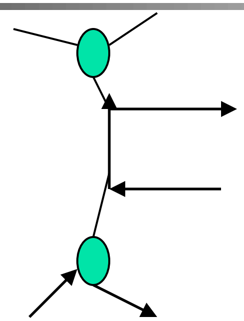
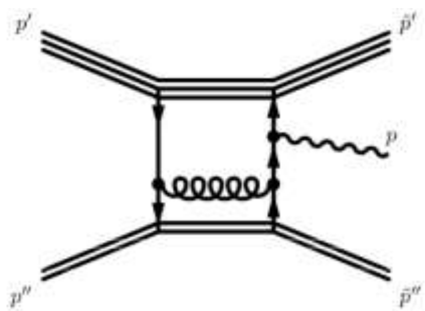
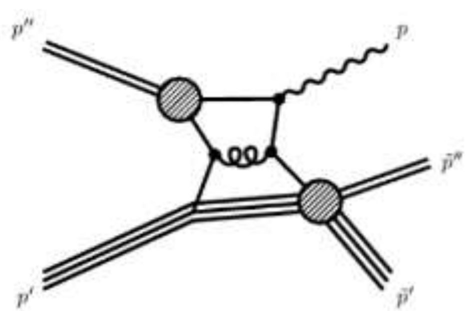
- (Nucleon GPD)x(pion Compton FF) – very forward region
- (Nucleon GPD)x(pion Compton GPD) – all x_F
- How to select? – interference with EM

(Nucleon FF)x(pion FF)

Interference effects

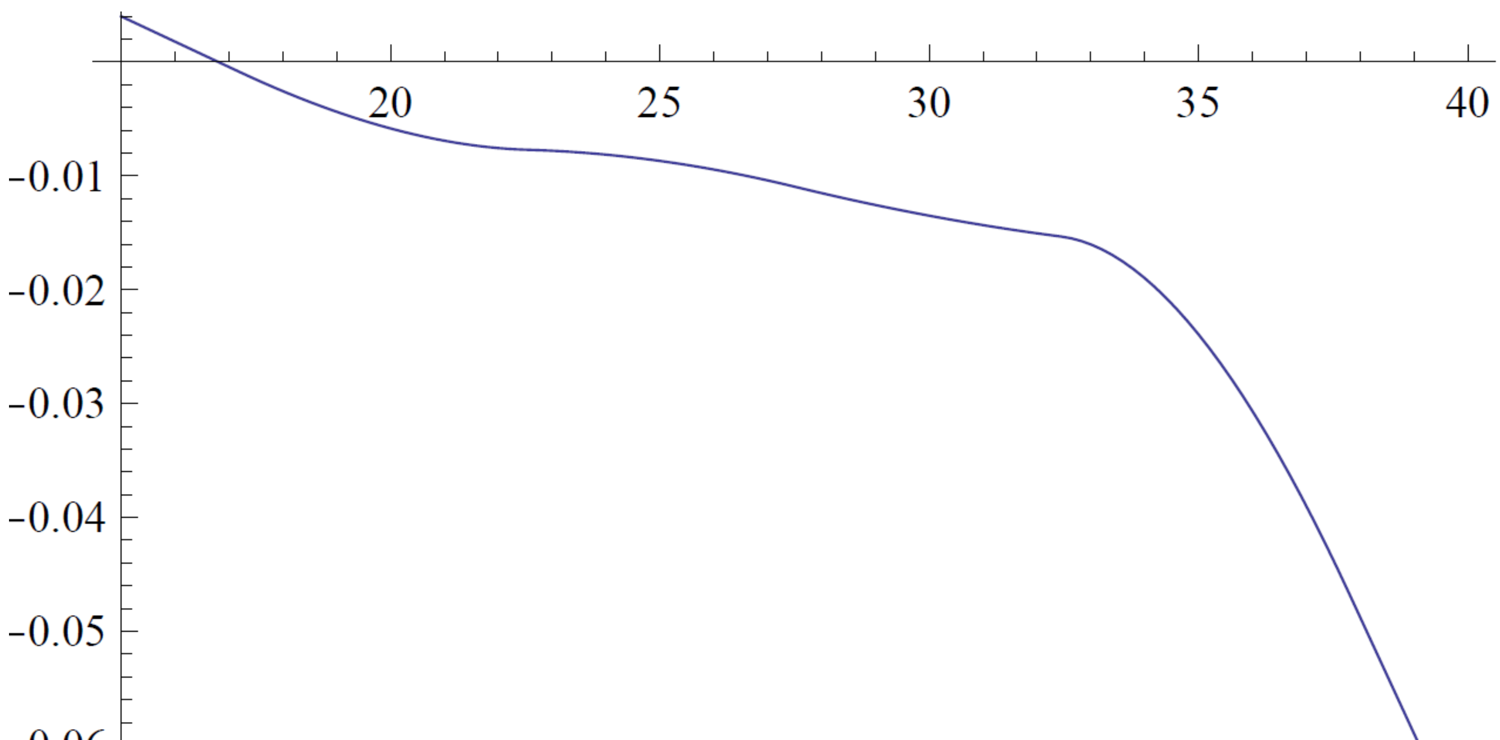
- Interference with pure EM (FFxFF) production of (C-even) lepton pair contains only real IR safe part of the amplitude and gives rise to charge asymmetry (Pivovarov,OT, work in progress)
- The way to extract GPDxGPD in central region from inclusive DY

Interference of EM, GPD and DD mechanism

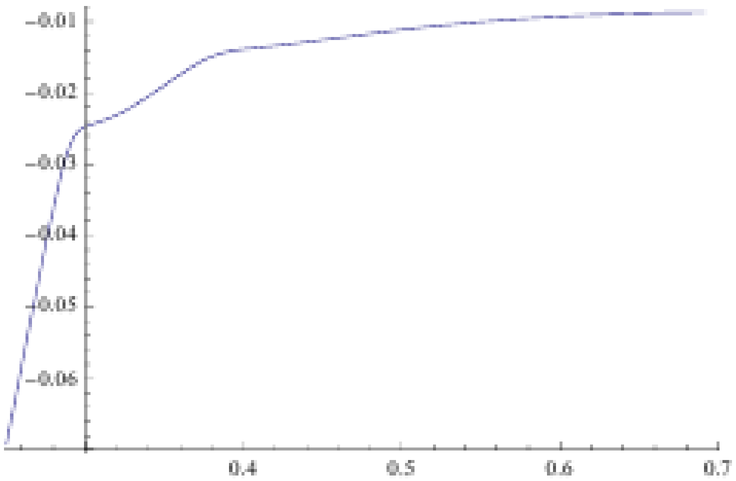
-  (2 diagrams)
-  (16 diagrams)
- (8 diagrams) 

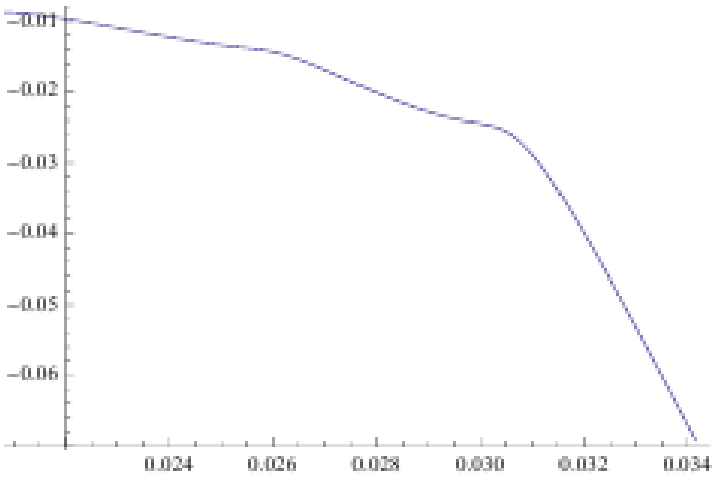
Interference with EM mechanism

- Charge asymmetry (muon-antimuon interchange) vs cm muon angle



(Anti)muon Lab frame asymmetry





Interference with DGP mechanism

- In the forward region: interference with BDP-like mechanism with TDA (Pivovarov,OT'14)

$$\frac{d\sigma}{\mathrm{d}^3\tilde{p}'\mathrm{d}^3\tilde{p}''} = \frac{\alpha_{(em)}\alpha^2_{(s)}}{2^6 N_c^4 \pi^2 \tilde{\varepsilon}' \tilde{\varepsilon}''} \cdot \frac{1}{(p', p'')^2} \cdot$$

$$\cdot \left[\frac{8s^2}{s_1 s_2} |I_1|^2 + \frac{8s}{s_1 + s_2 - s} |I_2|^2 + 4s \left(\frac{1}{s_2} + \frac{s}{s_1(s_1 + s_2 - s)} \right) (I_1^* I_2 + I_2^* I_1) \right] \cdot$$

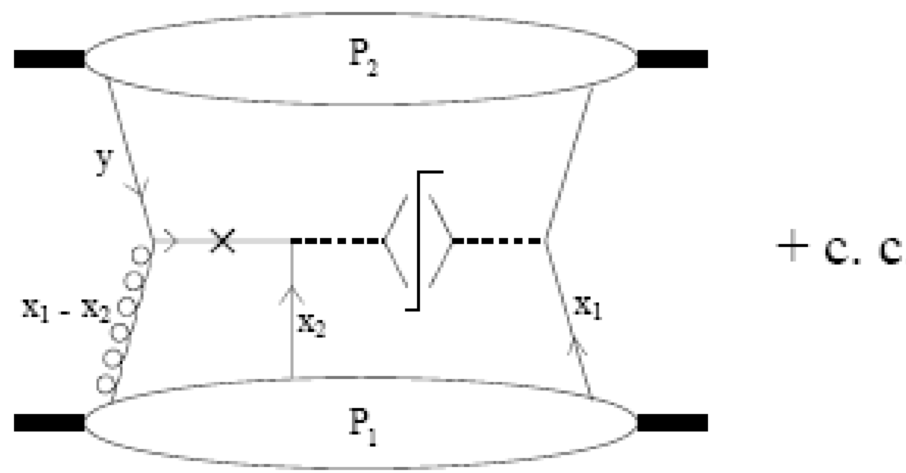
$$\cdot \delta(q^2 - m_\gamma^2)$$

SSA in DY

- TM integrated DY with one transverse polarized beam – unique SSA – gluonic pole (Hammon, Schaefer, OT) – "factor 2" problem

$$A = g\,\frac{\sin 2\theta\ \cos\phi \left[T(x,x) - x\frac{dT(x,x)}{dx}\right]}{M\left[1+\cos^2\theta\right] q(x)}$$

- Twist 3 – energy should not be too large(J-PARC)



Contour gauge in DY (Anikin,OT, **PLB690 (2010) 519; Direct photons - arXiv: 1501.05900**)

- Motivation of contour gauge – elimination of link $[-\infty^-, 0^-] = 1$

$$[-\infty^-, 0^-] = Pexp\Big\{-ig \int_{-\infty}^{0} dz^- A^+(0, z^-, \vec{0}_T)\Big\}$$

- Field

$$A^\mu(z) = \int_{-\infty}^{\infty} d\omega^- \theta(z^- - \omega^-) G^{+\mu}(\omega^-) + A^\mu(-\infty)$$

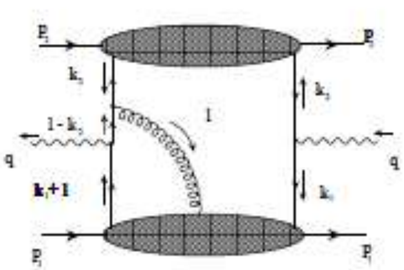
- Gluonic pole appearance $B^V(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon}$

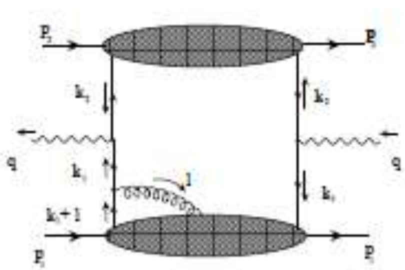
- cf naïve expectation $B^V(x_1, x_2) = \frac{\mathcal{P}}{x_1 - x_2} T(x_1, x_2)$

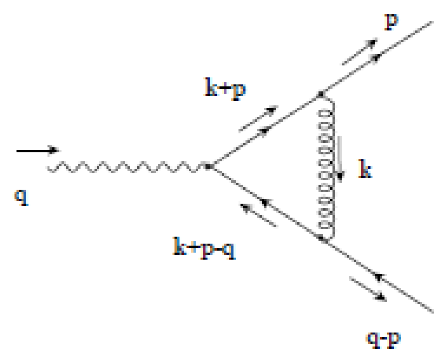
- Source of "Hidden" phase

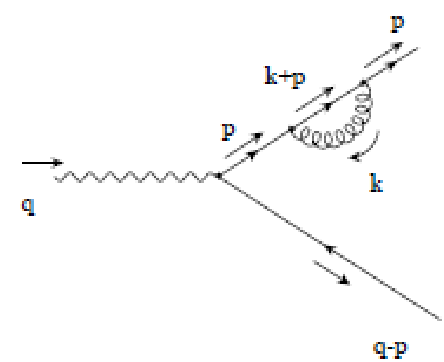
New phases and (EM) gauge invariance

- EM GI (experience from g2,DVCS) – 2 contributions









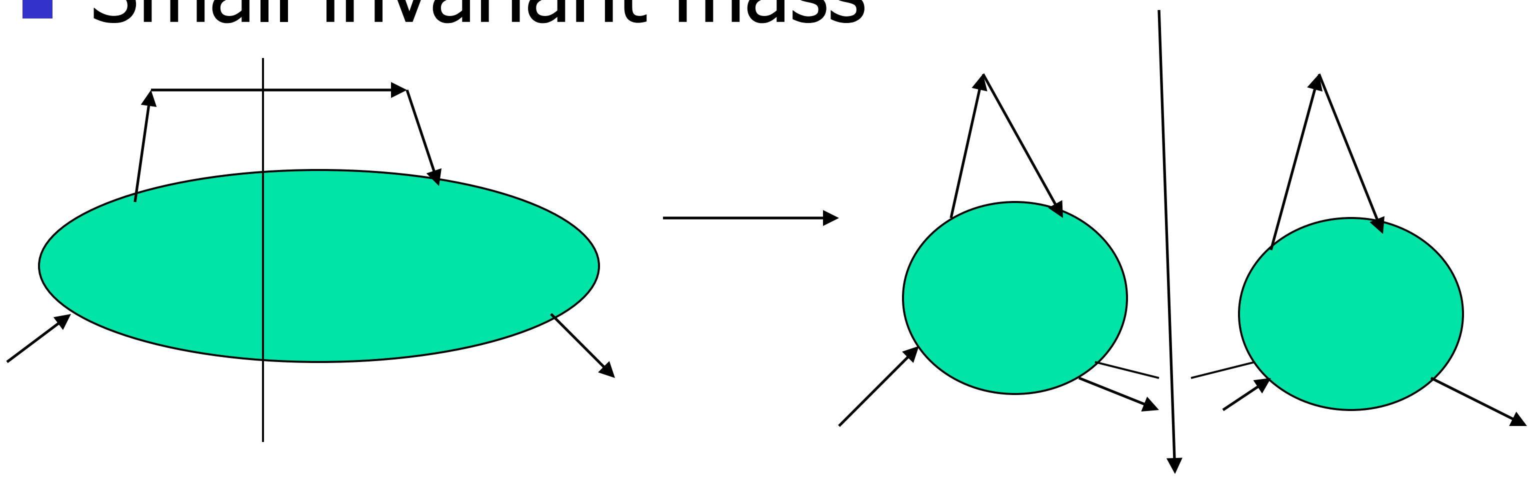
- Cf PT – only one of two diagrams contribute to SSA and required for GI
- NP tw3 analog - GI only if GP absent

Hidden phase

- Analogous to Beliitsky, Ji, Yuan finding for gauge link?! (GP/Sivers relation – Boer,Mulders,Pijlman; Ji, Qiu, Vogelsang, Yuan)
- Plasma physics: phase of naively real poles - Landau damping
- GP – “Hidden phases” – experimental tests@COMPASS?
- TM averaged DY with transverse polarized target

Exclusive limit : DIS and space-like (transitional and elastic) FFs

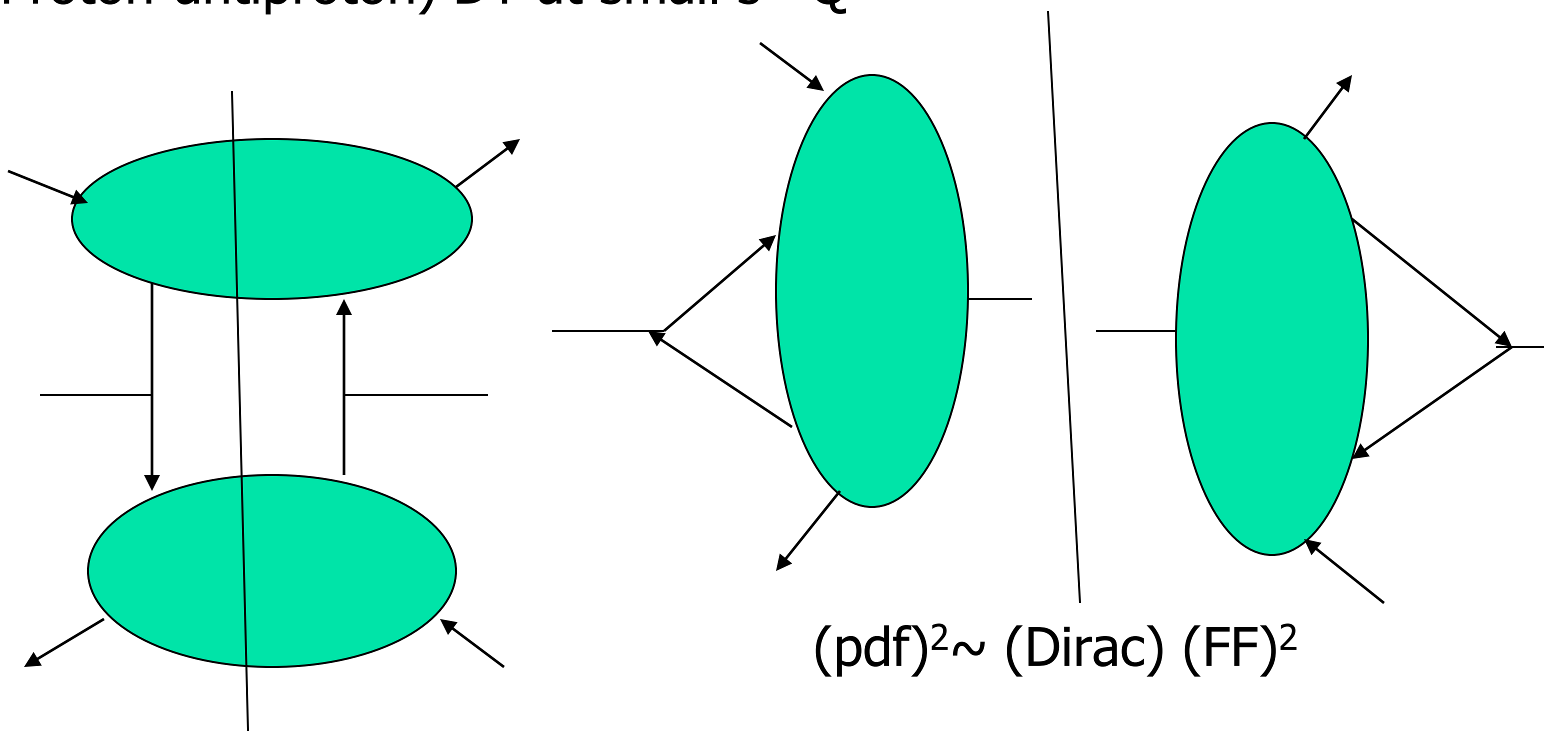
- Small invariant mass



- May be related to unitarity analyticity and DR (OT'05)
- Relation between x ->1 and large Q^2
- pdf ~ $(FF)^2$

Exclusive limit of DY and time-like FFs (OT'14)

- (Proton-antiproton) DY at small s - Q^2



- $(\text{pdf})^2 \sim (\text{Dirac})\ (\text{FF})^2$

- Other beams – baryon number conservation – time-like transition FFs

Comparing space-like and time-like FFs

- "Duality intervals" - from mass to x-space
- DIS: $(P+q)^2 = (P_f + \delta P_{DIS}))^2 = (M + \mu_{DIS})^2$ μ_{DIS} ~ pion related scale
- Deviation of $x_B(\equiv 1 - \delta_{DIS})$ from 1

$$\delta_{DIS} \sim 2M\mu_{DIS}/Q^2.$$

- DY: $(P_1 + P_2)^2 = (q + \delta P_{DY})^2$

- Deviation of $\tau = Q^2/s(\equiv 1 - \delta_{DY})$ from 1

$$\delta_{DY} \sim 2\mu_{DY}/Q$$

DR: FFs from duality intervals

- DIS: $F^2_{SL} \sim \int_0^{\delta_{DIS}} d\bar{x} f(\bar{x}) \quad x = 1 - \bar{x}$
- DY: $F^2_{TL} \sim \int_0^{\delta_{DY}} d\bar{x}_1 d\bar{x}_2 f(\bar{x}_1) f(\bar{x}_2) \delta(\delta_{DY} - \bar{x}_1 - \bar{x}_2)$
- Proton-antiproton DY –same parton distributions $f(\bar{x}) = C\bar{x}^a$

$$F^2_{SL}(Q^2) \sim \frac{C}{a+1}\left(\frac{2M\mu_{DIS}}{Q^2}\right)^{a+1} ; \; F^2_{TL}(Q^2) \sim \frac{C^2}{2(a+1)}\left(\frac{4\mu^2_{DY}}{Q^2}\right)^{a+1}$$

- Pion: a=1 supported !?

SL vs TL

- Same Q-dependence
- Normalization –defined by distribution scale (~5) and duality intervals
- Asymptotically coincide – scales close to QCDSR pion duality interval (rather than pion mass) similar (equal?!) for DIS and DY)!?

Sivers function and formfactors

- Relation between Sivers function and AMM known on the level of matrix elements (Brodsky, Schmidt, Burkardt)
- Phase?
- Duality for observables?

BG/DYW type duality for DY SSA in exclusive limit

- Proton-antiproton DY – valence annihilation – analyticity - cross section is described by Dirac FF squared
- The SSA (analyticity?!) similar to twist 3 one- due to interference of Dirac and Pauli FF's with a phase shift (Rekalo,Brodsky)
- Exclusive large energy limit; x -> 1 :
  $T(x,x)/q(x) \to \text{Im } F2/F1(Q^2 \sim M^2(1-x))$
- Both directions – estimate of Sivers at large x and explanation of phases in FF's

CONCLUSIONS/OUTLOOK

- GPDs meet DY in ExDY processes
- Unique role of interference and QCD induced charge asymmetry for lepton pairs production at COMPASS
- SSA@DY factor 2 (“Landau damping”) – can be tested at COMPASS?
- BG/DYW for FF’s – natural physical interpretation of Sivers function